Analysis of Position Tracking in Torque Control of Humanoid Robots Considering Joint Elasticity and Time Delay

Jaesung Jung\(^1\), Soonwook Hwang\(^1\), Yisoo Lee\(^1\), Jaehoon Sim\(^1\), and Jaeheung Park\(^{1,2}\)

**Abstract**—This study investigates the position tracking performance of torque controlled humanoid robots in the presence of joint elasticity and time delay in torque command. One of the main purposes using torque control for humanoid robots is to achieve compliant behaviors on uncertain external disturbance such as uneven terrain and interaction with human. On the other hand, high performance of position tracking is also required to implement motion control of robots. In this study, the effects of joint elasticity and time delay in torque command area investigated in terms of position tracking. First, a joint model is derived and validated, which reflects the elasticity and time delay. Frequency response analysis is exploited to theoretically evaluate the performance of the control system for position tracking. This joint model with the elasticity and time delay is used to estimate the limitations in the controller design for our torque controlled humanoid robot. Theoretical analysis and its comparison with experimental results demonstrate that the joint elasticity and time delay significantly affect the system performance.

I. INTRODUCTION

Torque control is one of the effective strategies for humanoid robots to operate in complex environments, facilitating their behaviors compliant with external disturbance [1], [2]. In contrast with position control that keeps each joint of the robots highly stiff, a wide range of the joint stiffness is possible in accordance with various situations in torque control. These advantages are beneficial to humanoid robots to safely carry out both manipulation and locomotion in human environments, where arbitrary disturbance is caused by unexpected contact or collision [3], [4].

Implementation of torque control to humanoid robots typically requires a central computer as a high level controller, and distributed motor drives as a low level controller which controls joint torque of actuators [5]. The central computer has a relatively slow control period [2], [6], due to a large amount of the computation required to solve the torque solution for motion and force control of the humanoid robot [7], [8]. The motor drive controls the single joint at a faster cycle than the central computer to generate the joint torque commanded by the high level controller. Generally, there exists a time delay in communication between the two controllers. The inevitable processing delay in motor drives deteriorates the control stability [9].

For most actuators used for robots, a gear reducer is installed to amplify the generated torque by the actuator [10]. Joint elasticity is induced by the deformable metallic material of the gear reducer such as harmonic drives [11], [12]. It is well known that the joint elasticity adversely affects the performance of feedback control for position tracking, which leads to oscillation [13], [14].

Fig. 1 is a configuration of the joint system of the robot, which reflects the intrinsic joint elasticity at the gear reducer as well as the time delay at motor drive. Furthermore, time delay in communication amplifies the adverse effect of the joint elasticity in the performance of torque control. The time delay usually occurs when the torque control is used, because the torque control is different from the position control. The typical position control commands reference value of position to the motor drive directly and the drive calculates and generates desired torque [15]. On the other hand, the torque control separates the torque calculation part and generation part, and the central computer and the motor drive respectively handle the calculation and generation.

To understand the limitation of torque control performance in terms of position tracking, a system that takes into account of the joint elasticity is theoretically analyzed. Modeling of the system also reflects the adverse influence of time delay in communication, of which the presence is experimentally verified. By combining the feedback controller for position tracking with the system model, the control performance is investigated through frequency response analysis. Validation of the analysis is achieved through experiments with both a single joint and a biped robot with high degrees of freedom (DoF). Through this study, we present a useful model to design controllers for torque controlled robots. By using presented model, we can explain the limitation of PD control. The PD control was chosen as an example in the paper to demonstrate the effectiveness of the model in the controller design, because it is commonly used for implementing position control. However, the proposed joint model is useful to design other control methods.

The remainder of this paper is organized as follows. In Section II, modeling of the system is discussed with taking account of both joint elasticity and time delay. Based on
the system model that includes the feedback controller, frequency response analysis is described in Section III, providing the information of the control performance. Experiment results are provided with feedback gains that assure stable control and the resultant tracking accuracy in Section IV, to demonstrate the model that includes joint elasticity and time delay. In Section V, conclusion is remarked with summary and future works.

II. SYSTEM MODELING

In order to analyze the control performance of humanoid robots, characterization of each joint, which is the basis of operation, should be preceded. Modeling of the joint system is conducted to take account of both time delay in communication and joint elasticity. Especially, these occur when each motor drive locally controls the motor and the torque input is remotely computed in the central computer.

The testbed is set up to verify the issues. It consists of a brushless DC (BLDC) electric motor from Parker Hannifin Corp., a harmonic drive with the gear ratio of 100:1, and a link of which length is 0.6 m and weight is 3.9 kg. A Linux-OS based computer with i7-3770 CPU and a motor drive from Elmo Motion Control Ltd. are used as the high level and low level controllers, respectively. Xenomai is utilized as a software framework for the real-time capability. The EtherCAT is utilized for the communication between the computer and the motor drive. Current control is enabled using CANopen protocols. Assuming that the input current of the motor is proportional to the output torque, torque control is performed using the current controller of the motor drive. The testbed utilized in the experiment is shown in Fig. 2.

A. Joint Elasticity

Considering only inertias of the motor and the load, the relationship between the input torque $\Gamma$ and output position $P$ is as follows.

$$ P = \frac{1}{J_M + J_L}s^2 \Gamma, \quad (1) $$

where $J_M$ is the motor inertia and $J_L$ is the load inertia connected to the motor.

In the system described in (1), the positions of motor ($P_M$) and load ($P_L$) are always the same ($P = P_M = P_L$). If there exists the joint elasticity, the relationship between the positions changes. Fig. 3 shows the block diagram of the elastic coupling between the motor and the load. Due to the elasticity, the motor and the load react differently and the differences of position and velocity occur. The position difference between the $P_M$ and the $P_L$ and the difference between the motor velocity ($V_M$) and load velocity ($V_L$) affect the input torque. The transfer function from the input torque $\Gamma$ and output motor position $P_M$ can be described as (2), where $K$ and $B$ are coefficients of the spring and damper, respectively [16].

$$ \frac{P_M}{\Gamma} = \frac{1}{(J_M + J_L)s^2 + Bs + K} \quad (2) $
Therefore, the feedback gain increasing is heavily affected by the motor inertia and that the influence of the load inertia is small.

B. Time Delay

Time delay is an inevitable phenomenon in communication, due to the interpretation of digital data to analog signals. When we use the torque control, the delay of 4~6 ticks is present. That is, when the robot is controlled at 2 kHz, the delay of about 2~3 ms occurs. The motor drives from other companies will also have the delay, but the size of delay may vary. The delay is identified by utilizing project data objects (PDO) of torque signals including the target torque, the demand torque, and the actual torque. The target torque is the torque we command to the motor drive, the demand torque, and the actual torque. The target torque is the torque we command to the motor drive, the demand torque is the desired torque that the motor drive will create, and the actual torque is the measured torque calculated by the measured current that the drive actually generates. Fig. 6 shows the delay between the target torque (\( \Gamma_t \)), the demand torque (\( \Gamma_d \)), and the actual torque (\( \Gamma_a \)) of the motor drive. Each value was recorded at 0.5 ms intervals.

The time delay of \( T \) seconds is represented as \( e^{-Ts} \). By multiplying \( e^{-Ts} \) to (2), we can obtain the transfer function which considers both the elasticity and the time delay. The time delay causes a phase lag, and the effect of the phase lag can be seen in the Bode plot (Fig. 7). As seen in Fig. 7, the gain margin is reduced because of the phase lag. Therefore, it can be seen that the time delay has a negative effect on the magnitude of feedback gains.

C. Modeling of a Joint Control System

Both elasticity and time delay affect the feedback gain, so both effects must be considered when modeling the single joint control system. A joint control system configuration for tracking the position is the following block diagram in Fig. 8. In this system, joint is controlled to track the desired position \( (P_d) \) with a position controller \( (D) \). The position controller can be any type of controller (e.g, PD, PID) and it calculates target torque for the joint control. The demand torque is transferred to the current controller \( C \) after the time delay \( T \). The current controller is modeled approximately as seen below,

\[
C = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}.
\]

The \( M \) is a model which describe the motor with the load including the elasticity. This system also reflects disturbance \( d \) and the measurement noise \( n \).

III. CHARACTERISTICS OF FEEDBACK SYSTEM

This section presents the analysis of PD control for position tracking and the analysis in operational space. By analyzing the controller, the adverse effects of the time delay and the elasticity on controllers will be shown. The analysis in operational space will show the relationship between the joint space feedback gains and the operational space feedback gains.

Fig. 4. Bode plot of the joint with and without the elasticity.

Fig. 5. Bode plot of joint with different load inertias.

Fig. 6. Target input torque, demand torque and actual torque from Elmo motor drive.

Fig. 7. Bode plot of joint with time delay and without time delay.
A. Frequency Analysis on PD control

With the joint model in Section II-C and the PD controller ($D = K_p + sK_v$), the position of motor is represented as follows.

$$P_M = \frac{CM(K_p + sK_v)}{1 + CM(K_p + sK_v)} P_d - \frac{CM(K_p + sK_v)}{1 + CM(K_p + sK_v)} n$$

$$+ \frac{CM}{1 + CM(K_p + sK_v)} d$$

There are limitations in increasing the feedback gains due to the time delay and the elasticity. This limitation can be known by the gain margin at the Bode plot. When the feedback gain becomes larger, the system will become unstable because there is lack of gain margin. Fig. 9 is the open loop Bode plot of block diagram of a joint system with the PD controller.

The effect of the elasticity and time delay on the gain margin can be seen in denoted points (a), (b), and (c) in Fig. 9. The elasticity reduces the gain margin from 52.1 dB to 27.5 dB, which is seen in Fig. 9 (a). As seen in Fig. 9 (b), the time delay affects to the gain margin to reduce from 52.1 dB to 31 dB. Fig. 9 (c) shows that the gain margin is reduced from 52.1 dB to 6.2 dB due to the both elasticity and time delay. It can be seen that the gain margin is greatly reduced when time delay and elasticity are present together.

As seen in Fig. 5, the magnitude of the low frequency decreases as the inertia of the load increases. Therefore, the feedback gain of the joint with the link of large inertia should be higher than the joint with the link of small inertia for similar control performance at the low frequency. However, due to the time delay and the elasticity, it is difficult to increase the feedback gain for the joint control sufficiently as the load inertia increases. Therefore, the joint with a large inertia load (e.g., ankle joint of the supporting leg of the humanoid robots) cannot achieve high tracking performance with a PD control scheme.

B. Analysis in Operational Space

Each joint of torque controlled humanoid robots has limitation because of the time delay and the elasticity. If multiple joints are coupled together, it is difficult to assume the performance limit, because of the cross coupled effect. When the robot is controlled in operational space, the feedback gains of the task space control can be understood by its relationship with the feedback gains in joint space.

In joint space, if the Coriolis, centrifugal, and gravity force are perfectly compensated, the PD control of the multiple joints can be written as

$$\Gamma = A\ddot{q} = A(K'_{pq}\dot{q}_e + K'_{rv}\dot{q}_e), \quad q_e = q_d - q$$

$$K'_{pq} = AK_{pq}$$

$$K'_{rv} = AK_{rv}$$

A is the joint space inertia matrix, $K'_{pq}$ is the proportional gain for the joint space control, and $K'_{rv}$ is the derivative gain. The term $\Gamma$ is the torque vector, $q_d$ is the desired joint position and $q$ is the actual joint position. If the robot is controlled in operational space [17], the control torque can be composed as follows, also assuming that the Coriolis, centrifugal, and gravity force are compensated.

$$\Gamma = J^T F$$

$$= J^T \Lambda(K'_{px}\dot{x}_e + K'_{vx}\dot{x}_e), \quad x_e = x_d - x$$

$$= J^T (K_{px}\dot{x}_e + K_{vx}\dot{x}_e)$$

$$K_{px} = \Lambda K'_{px}$$

$$K_{vx} = \Lambda K'_{vx}$$

Where $F$ represents the operational space force, $x_d$ is a desired task position, and $x$ is an actual task position. The term $J$ is the Jacobian for the tasks and $\Lambda$ is the operational space inertia matrix. The operational gain matrices are written as $K'_{px}$ and $K'_{vx}$. If $x_d - x$ is small, $x_d - x \simeq J(q_d - q)$. Then, the relation between the gains in joint space and operational space can be approximated as follows.
A. Setup

For the experiments in both joint and operational space control, the testbed with single joint in Section II is used to verify the theoretical feedback gain limit and the actual limit of the feedback gain of torque control with position feedback. A 12-DoF biped robot DYROS-Red [18] in Fig. 10 is utilized to measure the limitation of the performance of the operational space control framework, when there exist the joint elasticity and time delay.

\[
\begin{align*}
\Gamma & \simeq J^T \Lambda (K_p^c J q_e + K_q^c J q_e) \\
& \simeq (J^T \Lambda K_p^c J) q_e + (J^T \Lambda K_q^c J) q_e
\end{align*}
\]

(9)

\[
K_{pq} = A K_{pq}^c \simeq J^T \Lambda K_{pq}^c J
\]

\[
K_{vq} = A K_{vq}^c \simeq J^T \Lambda K_{vq}^c J
\]

(10)

Therefore, we can estimate the limits on the gains of the operational space control using this relationship if the limits in the joint space control are known. However, the limits in the joint space control are not easily computed because there are dynamic coupling terms. The gain matrices can be approximated to be diagonal using 1-DOF model of each joint but this approximation may not be accurate enough to represent the real system. Experimental results and discussion will be presented in the following section.

IV. EXPERIMENT RESULT

Following experiments were carried out to demonstrate the system model that reflects the joint elasticity and the time delay. Firstly, the system model is validated using an experimental setup with a single joint testbed. Then control performance of the robot with high DoFs is investigated with operational space control.

A. Setup

For the experiments in both joint and operational space control, the testbed with single joint in Section II is used to verify the theoretical feedback gain limit and the actual limit of the feedback gain of torque control with position feedback. A 12-DoF biped robot DYROS-Red [18] in Fig. 10 is utilized to measure the limitation of the performance of the operational space control framework, when there exist the joint elasticity and time delay.

B. Single Joint Control

This experiment uses a PD controller to show the limitation of torque control with position feedback. The testbed uses harmonic gear for the transmission and its spring constant is measured as $1.0 \times 10^4$ Nm/\(\text{rad}\). It is measured by fixing the output of the joint and applying torques. The inertia of motor is $0.2 \times 10^{-4} \text{ kg} \cdot \text{m}^2$ and the load-to-motor inertia ratio was 4. Theoretically, the feedback gains can be set as $K_p = 1600 \text{ Nm/\(\text{rad}\)}$ and $K_v = 77 \text{ Nm\cdots/\(\text{rad}\)}$. In this condition, gain margin is $76.8 \times 10^{-4} \text{ dB}$ at 494 rad/s. Fig. 11 shows the result of the experiment. The experiment compares two cases. One is the case that feedback gain was set as $K_p = 1600 \text{ Nm/\(\text{rad}\)}$ and $K_v = 80 \text{ Nm\cdots/\(\text{rad}\)}$. The feedback gain was set as $K_p = 1600 \text{ Nm/\(\text{rad}\)}$ and $K_v = 70 \text{ Nm\cdots/\(\text{rad}\)}$ which is lower than the theoretically set.

When the actual $K_v$ exceeds theoretical gain, the joint
TABLE I
THE LARGEST JOINT SPACE FEEDBACK GAINS CONVERTED FROM OPERATIONAL SPACE FEEDBACK GAINS DURING THE PD CONTROL EXPERIMENT.

<table>
<thead>
<tr>
<th></th>
<th>Hip Yaw</th>
<th>Hip Roll</th>
<th>Hip Pitch</th>
<th>Knee Pitch</th>
<th>Ankle Pitch</th>
<th>Ankle Roll</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Left Leg</strong></td>
<td>K_{py}</td>
<td>K_{ry}</td>
<td>K_{py}</td>
<td>K_{ry}</td>
<td>K_{py}</td>
<td>K_{ry}</td>
</tr>
<tr>
<td></td>
<td>4.180</td>
<td>4.731</td>
<td>4.216</td>
<td>12.281</td>
<td>12.334</td>
<td>91.32</td>
</tr>
<tr>
<td><strong>Right Leg</strong></td>
<td>K_{py}</td>
<td>K_{ry}</td>
<td>K_{py}</td>
<td>K_{ry}</td>
<td>K_{py}</td>
<td>K_{ry}</td>
</tr>
<tr>
<td></td>
<td>4.370</td>
<td>6.200</td>
<td>22.5</td>
<td>56.0</td>
<td>22.5</td>
<td>36.6</td>
</tr>
</tbody>
</table>

C. Operational Space Control of Biped Robot

**DYROS-Red** uses different motors and has different shaft design from 1-DOF test-bed, so the gains can be set higher. When each joint is controlled independently, gains of joints can be set to around \( K_{pqy} = 1.0 \times 10^4 \text{ Nm/ rad} \) and \( K_{vq} = 300 \text{ Nm s/ rad} \).

The task of the robot is controlled in operational space, using the operational space based whole-body control framework [19] with PD control. In this experiment, the position of the center of mass (CoM) is controlled to track the trajectory generated with cubic spline. Fig. 12 and 14 show the tracking performance when the CoM of the robot is controlled to translate 6.0 cm in x, y, and z directions simultaneously (total 10.4 cm) in 1.0 sec.

The gains used in operational space are \( K'_{px} = 50 \text{ s}^{-2}/\text{m}^2 \) and \( K'_{vy} = 1 \text{ s}^{-1}/\text{m}^2 \) for x direction, \( K'_{px} = 90 \text{ s}^{-2}/\text{m}^2 \) and \( K'_{vy} = 5 \text{ s}^{-1}/\text{m}^2 \) for y direction, and \( K'_{px} = 200 \text{ s}^{-2}/\text{m}^2 \) and \( K'_{vy} = 15 \text{ s}^{-1}/\text{m}^2 \) for z direction. The gains are set to the values that are stable and can maximize performance of tracking. These gains are empirically derived from the repetitive experiments. These gains (\( K'_{pqy} \) and \( K_{vq} \)) can be converted to the joint space gains (\( K_{pqy} \) and \( K_{vq} \)) through the method in Section III-B. The diagonal elements of converted result are shown in the Table I. From the gains in Table I, we can speculate the gain limit of joints. Especially, D gains of Knee Pitch and Ankle roll of both legs are much larger than other joints. This shows that these two joints of each leg are problematic in control stability.

However, the values in Table I are smaller than the gain limit of each joint. The reason for this gap may be due to the dynamic coupling that comes from the off-diagonal elements in \( K'_{pqy} \). When the operational space gains are converted into the joint space gain, gains are multiplied by inertia matrix and Jacobian. So, even if \( K_{pqy} \) is a diagonal matrix, \( K_{pqy} \) has off-diagonal elements. Therefore, we believe that this coupling effect can be the reason why the diagonal components of the gains cannot be set as high as the gains for the case of single joint control.

In this experiment, P/PI cascade controller was designed as an example to improve the performance of the operational space control. The structure of P/PI controller is shown in Fig. 13. Fig. 14 shows the result when P/PI controller is applied to perform the same task with the case of using PD controller. P/PI used the gains as \( K_p = 40 \text{ s}^{-1}, K_v = 1 \text{ m}^{-1}/\text{s} \), and \( K_p = 1 \text{ s}^{-1} \) for x direction, \( K_p = 20 \text{ s}^{-1}, K_v = 5 \text{ m}^{-1}/\text{s} \), and \( K_p = 0.1 \text{ s}^{-1} \) for y direction, \( K_p = 100 \text{ s}^{-1}, K_v = 5 \text{ m}^{-1}/\text{s} \), and \( K_p = 1.0 \text{ s}^{-1} \) for z direction. Gains are set based on the PD control gains. The steady state errors and tracking errors in all directions were reduced, compared to those of PD controller. Especially, x direction steady state error was reduced from 14.0 mm to 5.6 mm. In y direction, the steady state error was reduced from 5.1 mm to 2.9 mm and the z direction steady state error was reduced from 13.0 mm to 1.0 mm. Because of the integral term, P/PI can reduce steady state error. The integral term compensates the low frequency magnitude which is caused by the load.

There are several limitations in the results, even using P/PI controller. Firstly, it is challenging to obtain the consistent re-
sponse depending on the task directions. The effect of friction is different depending on the direction. Also, depending on the direction of movement, the inertia continues to change, and hence the gains applied to each joint always change.

Secondly, although the steady-state error is reduced by the influence of the integral term in P/PI controller, there is relatively less decrease in transient error compared to steady-state error. Also there still exists a fluctuation during the transition. This fluctuation may be due to the stick-slip phenomenon induced by the nonlinear joint friction in low speed [20].

V. CONCLUSIONS

The objective of this study is to understand the limitations of torque controlled humanoid robots in two factors, the joint elasticity and time delay. The model with the joint elasticity and time delay is useful to understand and design the controller for humanoid robots. To enhance the position tracking performance, the feedback gain should be increased to the high level, but the elasticity and the time delay limit the feedback gain. Influence of the joint elasticity and the time delay in torque control are verified with theoretical analysis as well as experimental validation. It is notable that time delay amplifies the adverse effect of joint elasticity in terms of the reduction in gain margin. Without one of these factors, the adverse effects are far less than when both factors are present.

Our study is expected to be utilized to analyze and determine the available control gains for the tasks of robots with high DoFs, where the joint elasticity and the time delay exist. Since the gain tuning of most humanoid robots is performed through trial and error, it takes much time and effort in the process. Our research will allow the selection of the reasonable feedback gains in high-DoF robots and reduce the time required for gain tuning.

The humanoid robot changes the contact state while performing complex tasks including walking. As the contact state changes, the load-to-motor inertia ratio of each joint changes. Typically, the load-to-motor inertia ratio of the ankle joint in the supporting leg is significantly increased, compared to the double support phase. Due to this reason, the limited performance caused by the elasticity and the time delay makes it challenging for torque controlled humanoid robots in a single support phase, which is essential for the bipedal walking control. Therefore, in order to achieve the desirable control performance, it is required to determine the feedback gain according to the contact state of the robot.

In the future, the estimation of the gains in the operational space from those in joint space, considering the cross-coupling effect, will be investigated. Then, we will be able to develop a real-time gain determination method online, according to the state of the robot. Also, we would like to investigate advanced controllers that can achieve desired performance.

ACKNOWLEDGEMENT

This work was supported by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MSIP) (No. NRF2015R1A2A1A10055798) and the Technology Innovation Program (No. 10060081) funded by the Ministry of Trade, Industry & Energy (MI, Korea).

REFERENCES