Screw Motion and Its Application

GSCST, Intelligent Convergence Systems

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Dynamic Robotic Systems Lab

Definition

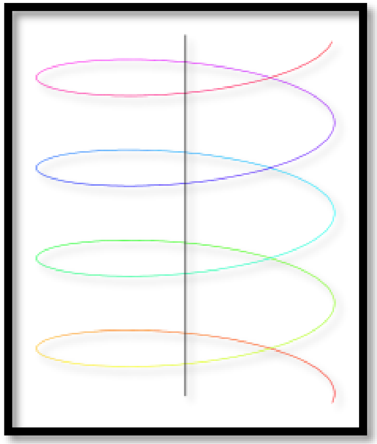
Application

Some Theorem

Example

Screw (Geometrical view)

Screw is rotation about axis to translation along that axis.



So we can calculate some point by using rotation axis(3-d) and pitch(1-d).
(Tip. It is similar to DH-parameter. DH-parameter has 4 scalars.)

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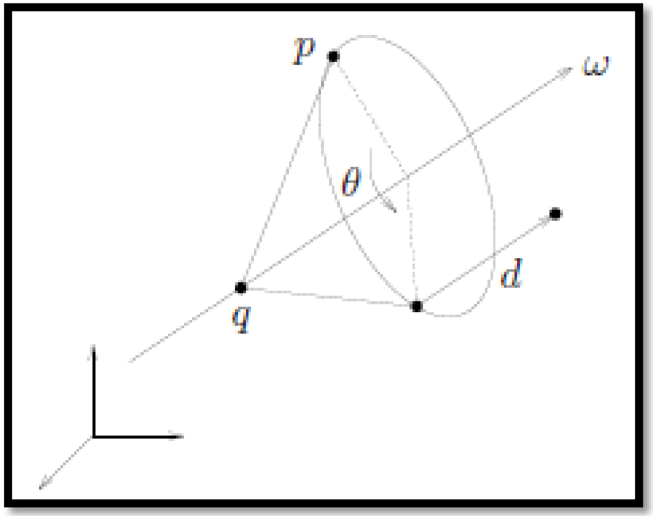
Screw (Robotics)

Consider a rigid body motion which consists of rotation about an axis in space through an angle (θ), followed by translation along the same axis by an amount d.

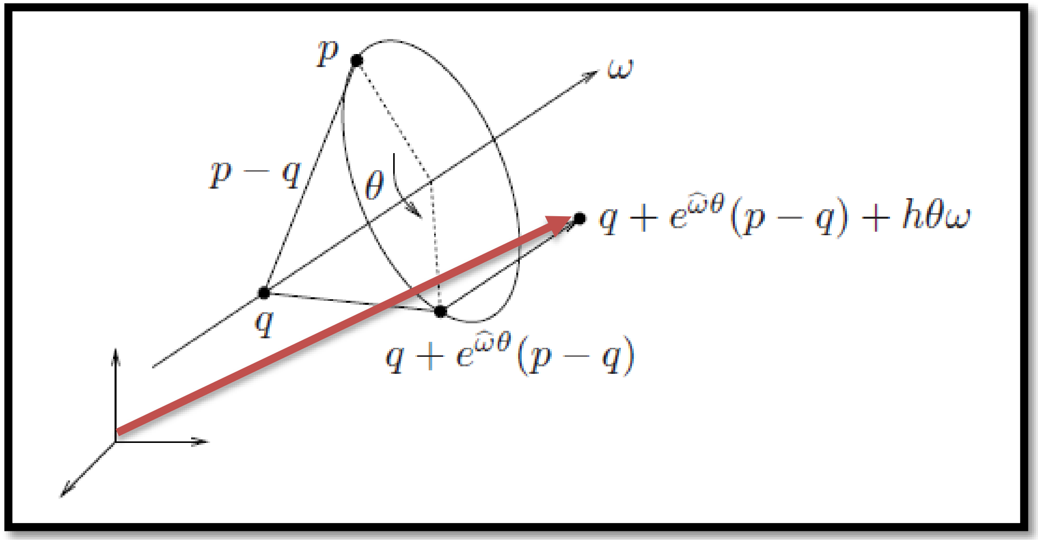


Definition. Screw Motion

Rotation about an axis by θ followed by translation about the same axis.

Screw Motion

To compute the rigid body transformation associated with a screw, we analyze the motion of a point p(red line).



The final location of the point is given by

$$p = q + e^{\omega\theta}(p - q) + h\theta\omega \text{ (vector form)}$$

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Screw Motion

$e^{\omega\theta}$ is representation of rotation matrix.

$$e^{\widehat{\omega}\theta} = I + \widehat{\omega}\sin\theta + \widehat{\omega}^2(1-\cos\theta)$$
$$= \begin{bmatrix} 1 - v_\theta(\omega_2^2+\omega_3^2) & \omega_1\omega_2 v_\theta - \omega_3 s_\theta & \omega_1\omega_3 v_\theta + \omega_2 s_\theta \\ \omega_1\omega_2 v_\theta + \omega_3 s_\theta & 1 - v_\theta(\omega_1^2+\omega_3^2) & \omega_2\omega_3 v_\theta - \omega_1 s_\theta \\ \omega_1\omega_3 v_\theta - \omega_2 s_\theta & \omega_2\omega_3 v_\theta + \omega_1 s_\theta & 1 - v_\theta(\omega_1^2+\omega_2^2) \end{bmatrix}$$
$$= \begin{bmatrix} \omega_1^2 v_\theta + c_\theta & \omega_1\omega_2 v_\theta - \omega_3 s_\theta & \omega_1\omega_3 v_\theta + \omega_2 s_\theta \\ \omega_1\omega_2 v_\theta + \omega_3 s_\theta & \omega_2^2 v_\theta + c_\theta & \omega_2\omega_3 v_\theta - \omega_1 s_\theta \\ \omega_1\omega_3 v_\theta - \omega_2 s_\theta & \omega_2\omega_3 v_\theta + \omega_1 s_\theta & \omega_3^2 v_\theta + c_\theta \end{bmatrix}.$$

(Tip. It is same as equivalent axis representation)

$p = q + e^{\omega\theta}(p-q) + h\theta\omega$ (vector form)

In homogenous form is given by,

$$\begin{bmatrix} p \\ 1 \end{bmatrix} = \begin{bmatrix} e^{\omega\theta} & (I - e^{\omega\theta})q + h\theta\omega \\ 0 & 1 \end{bmatrix} \begin{bmatrix} p \\ 1 \end{bmatrix}$$ (homogenous form)

Transformation matrix

Screw Motion

if we choose translation velocity,

$$v = -\omega \times q + h\omega$$

we can obtain transformation matrix by screw theory.

$$e^{\xi\theta} = \begin{bmatrix} e^{\omega\theta} & (I - e^{\omega\theta})(\omega \times v) + \omega\omega^T v\theta \\ 0 & 1 \end{bmatrix}$$

$$\xi = \begin{bmatrix} v \\ w \end{bmatrix} \in R^6$$

Chasles' Theorem

Every rigid body motion can be realized by a rotation about an axis combined with a translation parallel to that axis.

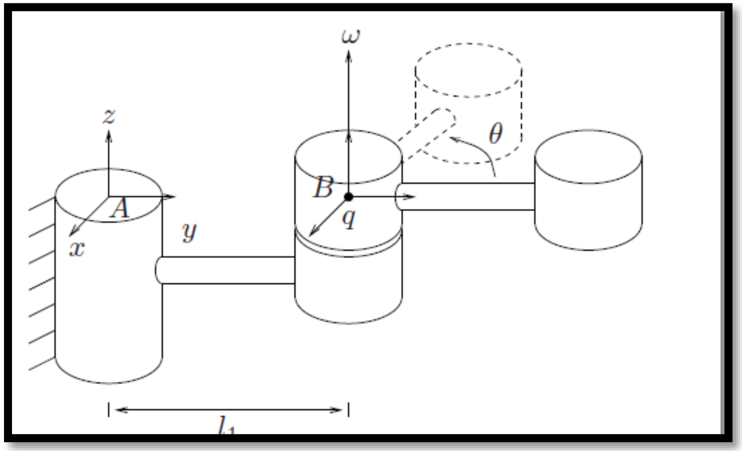
That's why we use screw theory.

An important result of screw theory is that geometric calculations for points using vectors have parallel geometric calculations for lines obtained by replacing vectors with screws. Based on screw theory, an efficient approach has also been developed for the type synthesis of parallel mechanisms.

Example. Rotation about a line.

Consider the motion of a rigid body rotating about a fixed axis in space. This motion corresponds to a zero-pitch screw about an axis in the ω=(0,0,1), and $q = (0, l_1, 0)$.

(Tip. In revolute joint, There is no pitch(h))



Example. Rotation about a line.

The Corresponding screw is

$$\xi = \begin{bmatrix} -\omega \times q \\ \omega \end{bmatrix}$$

Therefore, Homogeneous Transform matrix is

$$e^{\xi\theta} = \begin{bmatrix} e^{\omega\theta} & \left(I - e^{\omega\theta}\right)(\omega \times v) \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta & -\sin\theta & 0 & l_1 \sin\theta \\ \sin\theta & \cos\theta & 0 & l_1(1 - \cos\theta) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

It is same result homogeneous transform ${}^{B}_{A}T$ obtained from DH-parameter.

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Reference

1. Book: A Mathematical introduction to robotic manipulation (Murray)
2. Lecture Note: Robot Mechanism (Dongjun Lee, SNU)
3. Homepage: http://en.wikipedia.org/wiki/Screw_theory
4. Book: Geometry and Screw Theory for robotics (Stefano)