Whole-body Control of Non-holonomic Mobile Manipulator Based on Hierarchical Quadratic Programming and Continuous Task Transition

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Abstract—Whole-body manipulation for a non-holonomic mobile manipulator has potential in various fields including service and industrial robotics because it can provide a great deal of manipulation capability and mobility. This paper presents the whole-body controller of the non-holonomic mobile manipulator for achieving complex tasks effectively. The proposed controller is designed based on Hierarchical Quadratic Programming (HQP). Thus, the controller can deal with both equality and inequality tasks. Moreover, by applying the continuous task transition strategy, our controller can compute the continuous control input when the priorities of the tasks are changed. The proposed control scheme was implemented on a mobile base with a 7 DoFs robotic arm and its performance was validated during various experimental scenarios.

I. INTRODUCTION

Non-holonomic wheeled mobile manipulators have been developed in various fields such as disaster area and daily-life assistance because the workspace of the robot can be extended [1]–[4]. However, controlling the non-holonomic mobile manipulator is still a challenging issue because this robot is strong coupling system consisting of a mobile base and a manipulator with high Degrees of Freedom (DoFs). Therefore, a control framework of a non-holonomic mobile manipulator has been actively studied to generate whole-body motions to treat complex tasks hierarchically.

As the analytical approach, Padois et al. proposed a unified motion and force controller with the generalized inverse of Jacobian for the mobile manipulator in operational space [5]. Thanks to the null-space projection of the task space, this control framework can treat a predefined Stack of Tasks (SoTs). Similarly, White et al. proposed a task and null-space dynamic controller in operational space [6]. In [7], a whole-body impedance control was proposed by using the admittance control of the mobile platform. Although these controllers can treat the predefined SoTs with equality tasks, they cannot treat inequality tasks.

On the other hand, Model Predictive Controller (MPC) and the redundancy optimization based on Nonlinear Programming (NLP) were applied to the mobile manipulator [8]–[10]. Although these methods can handle inequality tasks as well as equality tasks, they cannot treat prioritized SoTs strictly. In addition, depending on the SoTs, there is a possibility that a feasible solution cannot be obtained.

In this paper, we propose a novel whole-body controller for the non-holonomic mobile manipulator based on Hierarchical Quadratic Programming (HQP). HQP is a cascade form of the Quadratic Programming (QP) to treat inequality tasks as well as equality tasks of prioritized SoTs [11], [12].

Thus, the main contributions of the proposed control framework for the non-holonomic mobile manipulator are as follows: First, the proposed control framework based on the HQP formulation can handle the prioritized tasks in SoTs strictly. In particular, the proposed controller for whole-body manipulation of the non-holonomic mobile manipulator can deal with inequality tasks as well as equality tasks. Second, the proposed framework can calculate continuous control input when the priorities of the tasks are changed during operation. The continuous task transition method guarantees the stable execution of the robot when arbitrary tasks are
inserted, removed, and swapped to the controller. Finally, the proposed controller was implemented on the four-wheeled mobile base with the 7-DoF manipulator in real-time, and its performance was demonstrated by experiments, as shown in Fig. 1.

The remainder of this paper is as follows. First, Sec. II introduces the system modeling, HQP-based controller for the non-holonomic mobile manipulator, and its continuous task transition method. Sec. III describes the experimental validations of the proposed control scheme. Finally, the paper is concluded in Sec. IV.

II. PROPOSED CONTROLLER DESIGN

In this section, we explain a system modeling for the non-holonomic mobile manipulator and its controller based on HQP with the continuous task transition algorithm. For enhancing the readability of this paper, the notations are listed in Table I.

A. Dynamics for Non-holonomic Mobile Manipulator

As shown in Fig. 2, let us consider the non-holonomic mobile manipulator with 2 wheels and \( n_w \)-DoFs manipulator. Since the floating base of mobile manipulator includes 3-DoFs of rigid body motion, the generalized coordinates can be represented as

\[
q = \begin{bmatrix} \zeta^T & q_b^T & q_m^T \end{bmatrix}^T,
\]

where \( \zeta = [x_c \ y_c \ \phi]^T \in \mathbb{R}^3 \) is the coordinate of the center of the mobile base, \( q_b = [\theta_r \ \theta_l]^T \in \mathbb{R}^2 \) is the joint angle vector for the robotic arm.

Assuming the kinematic constraint of the non-holonomic system and the non-slip condition of rolling direction of wheels, we can set the constraint matrix between rigid body motion of the mobile base and the generalized coordinates to satisfy the following equation.

\[
A(q)\dot{q} = 0,
\]

where \( A(q) \in \mathbb{R}^{3 \times (5 + n_m)} \) is the full-ranked constraint matrix [6], [14]. For other types of mobile base subject to non-holonomic constraint, the form of \( A(q) \) can be different in [5]. Using the null-space of \( A(q) \), we can obtain the following transform equation,

\[
\dot{q} = S(q)\eta,
\]

where \( S(q) \in \mathbb{R}^{(5 + n_m) \times (2 + n_m)} \) is the matrix satisfying \( A(q)S(q) = 0 \) and \( \eta = [\dot{q_b}^T \ \dot{q_m}^T]^T \in \mathbb{R}^{2 + n_m} \) is the joint velocity for actuators of the robot. Using this equation,

\[
\dot{x}_d = J_1 \eta + \hat{J}_1 \eta,
\]
With the system modeling of (6), a conventional QP formulation for the non-holonomic mobile manipulator to deal with $\mathcal{T}_1$ can be expressed as,

$$
\begin{align*}
\min_{\eta, u, w_1} & \quad \|x_{d_1} - \hat{J}_1 \eta - \dot{\hat{J}}_1 \eta\|_2, \\
\text{s. t.} & \quad \hat{M} \dot{\eta} + \hat{C} \eta + \tilde{g} = u + S^T \tau_{\text{ext}} \\
& \quad x_{d_1} = \hat{J}_1 \eta + \dot{\hat{J}}_1 \eta + w_1
\end{align*}
$$

If $\mathcal{T}_1$ has a feasible solution area when the cost of (8) is zero. Otherwise, the cost is greater than zero.

Thus, by replacing the object cost with the slack variable, $w_1$, (8) can be rewritten as

$$
\begin{align*}
\min_{\eta, u, w_1} & \quad \|w_1\|_2, \\
\text{s. t.} & \quad \hat{M} \dot{\eta} + \hat{C} \eta + \tilde{g} = u + S^T \tau_{\text{ext}} \\
& \quad \ddot{x}_{d_1} = \hat{J}_1 \eta + \dot{\hat{J}}_1 \eta + w_1
\end{align*}
$$

where $w_1 \in \mathbb{R}^{n_1}$ is a slack variable, which relaxes the infeasible solution area in $\mathcal{T}_1 \in \mathbb{R}^{n_1}$. Thus, even if there is no feasible solution of $\mathcal{T}_1$, (9) is always solved because $w_1$ becomes a zero vector. Note that the optimal slack variable, $w_1$, is the zero vector when $\mathcal{T}_1$ has a feasible solution area. Consequently, (9) is equivalent as (8). Likewise, the QP formulation with a single inequality task using the slack variable is as follow:

$$
\begin{align*}
\min_{\eta, u, w_1} & \quad \|w_1\|_2, \\
\text{s. t.} & \quad \hat{M} \dot{\eta} + \hat{C} \eta + \tilde{g} = u + S^T \tau_{\text{ext}} \\
& \quad \ddot{x}_{d_1} = \hat{J}_1 \eta + \dot{\hat{J}}_1 \eta + w_1 \leq \ddot{x}_{d_1}
\end{align*}
$$

Now, let us consider the case with two inequality tasks with priorities ($\mathcal{T}_1 \prec \mathcal{T}_2$). Extending (10), the HQP formulation which is the cascade of the QP problem with the slack variables is represented as

$$
\begin{align*}
\min_{\eta, u, w_2} & \quad \|w_2\|_2, \\
\text{s. t.} & \quad \hat{M} \dot{\eta} + \hat{C} \eta + \tilde{g} = u + S^T \tau_{\text{ext}} \\
& \quad \ddot{x}_{d_2} = \hat{J}_2 \eta + \dot{\hat{J}}_2 \eta + w_2 \leq \ddot{x}_{d_2} \\
& \quad \ddot{x}_{d_1} = \hat{J}_1 \eta + \dot{\hat{J}}_1 \eta + w_1 \leq \ddot{x}_{d_1}
\end{align*}
$$

where $w_2 \in \mathbb{R}^{n_2}$ is a slack variable of $\mathcal{T}_2 \in \mathbb{R}^{n_2}$ and $w_1^* \in \mathbb{R}^{n_1}$ is the predefined slack variable by solving (10). Thus, the solution of (11) strictly satisfies priority, $\mathcal{T}_1 \prec \mathcal{T}_2$, because this solution is in the feasible solution area of (10), as shown in Fig. 3.

Finally, the general HQP formulation of the non-holonomic mobile manipulator with $\mathcal{T}_1 \prec \ldots \prec \mathcal{T}_k$ can be defined as follows:

$$
\begin{align*}
\min_{\eta, u, w_k} & \quad \|w_k\|_2, \\
\text{s. t.} & \quad \hat{M} \dot{\eta} + \hat{C} \eta + \tilde{g} = u + S^T \tau_{\text{ext}} \\
& \quad \ddot{x}_{d_k} = \hat{J}_k \eta + \dot{\hat{J}}_k \eta + w_k \leq \ddot{x}_{d_k}
\end{align*}
$$

where $w_k^*$ is the optimal slack variable which is obtained by the HQP formulation with $\mathcal{T}_1 \prec \ldots \prec \mathcal{T}_{k-1}$. To calculate the feasible solution space of $\mathcal{T}_1 \prec \ldots \prec \mathcal{T}_k$ using (12), QP operations need $k$ times.

### C. Continuous Task Transition

The HQP-based controller for the non-holonomic mobile manipulator in Sec. II-B can deal with the predefined SoTs strictly. However, to perform complex tasks effectively, rescheduling of SoTs in real-time is necessary depending on situations. However, aforementioned sudden task transitions may generate discontinuous control input of the robot, which can adversely affect the stability and durability of the robots [15]–[17].

Let us recall the example of (10) and (11). When the transition from the single task, $\mathcal{T}_1$, of (10) to the prioritized tasks, $\mathcal{T}_1 \prec \mathcal{T}_2$, of (11) occurs, the discontinuity of the feasible solution exists, as shown in Fig. 3(a) and 3(b).

To solve this issue, we applied our previous continuous task transition approach [13] in the proposed controller of (12). The basic idea of this approach is to linearly interpolate the feasible solution areas between the previous SoTs and the current SoTs by using the activation parameter. Depending on the transition phase, the activation parameter has a value between 0 and 1 to modify the effect of the previous SoTs and the inequality bound of the new task.

In the following subsections, we describe how to insert, remove, and swap arbitrary tasks without the discontinuity of the solution.
1) Inserting and removing task: Let us consider the transition from (10) to (11). That is, $F_2$ is inserted as a lower priority task in SoT $F_1$. Using the activation parameter, $\beta$, the proposed task transition method of the example is expressed as

$$\min_{\eta, u, w_2} \|w_2\|_2,$$

subject to

$$M\dot{\eta} + C\eta + g = u + S^T \tau$$

$$\beta(\ddot{x}_d - \dot{j}_d) \leq \dot{j}_I \eta - (1 - \beta) \dot{j}_2 \eta^*_I + w_2 \leq \beta(\ddot{x}_d - \dot{j}_d)$$

$$\ddot{x}_1 \leq \dot{j}_I \eta + \dot{j}_I \eta + w_1 \leq \ddot{x}_1$$

where $\eta^*_I$ is the solution of (10). Thus, when $\beta = 0$, the solution of (13) is the same as that of (10), as shown in Fig. 4(a). If $\beta = 1$, (13) and (11) are perfectly equivalent. In addition, when $\beta$ increases from 0 to 1 (see the red line in Fig. 5), the feasible solution of (13) can be derived by interpolating the solution between (10) and (11) because the inequality bound of $F_2$ is modified by the $\beta$, as shown in Fig. 4(b).

On the other hand, if $F_1$ is removed in the SoTs, $F_1 \prec F_2$, we can also use (13). The only different thing is that $\beta$ is decreased to 0 during the transition phase, as shown in the blue line of Fig. 5.

2) Swapping tasks: Now, we explain how to rearrange tasks from $F_1 \prec F_2$ to $F_2 \prec F_1$. Similar to the approach of (13), the continuous task transition for swapping can be expressed as follow:

$$\min_{\eta, u, w_2} \|w_2\|_2,$$

subject to

$$M\dot{\eta} + C\eta + g = u + S^T \tau$$

$$\beta_1(\ddot{x}_d - \dot{j}_d) \leq \dot{j}_I \eta - (1 - \beta_2) \dot{j}_2 \eta^*_I + w_2 \leq \beta_2(\ddot{x}_d - \dot{j}_d)$$

$$\beta_1(\ddot{x}_d - \dot{j}_d) \leq \dot{j}_I \eta - (1 - \beta_1) \dot{j}_1 \eta^*_I + w_1 \leq \beta_1(\ddot{x}_d - \dot{j}_d)$$

where $\beta_1$ and $\beta_2$ are the decreasing and increasing activation parameters, respectively. Here, $\eta^*_I$ and $\eta^*_I$ are the solutions of $F_1 \prec F_2$ with $\beta_1$ and $F_2 \prec F_1$ with $\beta_2$, respectively.

It is worthwhile to note that the feasible solution of (14) with $\beta_1 = 0$ and $\beta_2 = 1$ is equivalent to that of the following equation with $F_2 \prec F_1$:

$$\min_{\dot{q}, \tau, w_1} \|w_1\|_2,$$

subject to

$$M\ddot{q} + C\dot{q} + g = \tau$$

$$\ddot{x}_1 - \dot{j}_1 \eta \leq \dot{j}_1 \eta + w_1 \leq \ddot{x}_1 - \dot{j}_1 \eta$$

$$\ddot{x}_2 - \dot{j}_2 \eta \leq \dot{j}_2 \eta + w_2 \leq \ddot{x}_2 - \dot{j}_2 \eta$$

Consequently, the proposed controller with the continuous task transition can treat the dynamically changing SoTs without the discontinuity of the control input.

III. EXPERIMENTS

The proposed control framework was validated through experiments with the non-holonomic mobile base with the 7-DoF robotic manipulator. The source code for Ubuntu 16.04 is available at [18] and more experimental results are described in [19]. The subsections below describe the details of our system specification and experimental results with the real robot.

A. System Overview

As shown in Fig. 6, the mobile manipulator for experiments consists of two parts: the velocity controlled four-wheel mobile base, Husky (Clearpath Robotics Co.) and 7-DoFs robot arm manipulator, Panda (Franka Emika Co.). In this mobile base, there is the ROS interface of the low-level controller in real-time and localization algorithm using wheel odometry\(^1\). The localization algorithm provides position and velocity of the center of the mobile base with respect to initial pose. The control frequencies of each manipulator and the base are 1 kHz and 10 Hz, respectively. Finally,

\(^1\)http://wiki.ros.org/robots/husky
the specification of the computer for the controller is i7 4.2 GHz with 16 GB RAM.

On the other hand, Fig. 7 shows our system structure with the mobile manipulator. Because our mobile base supports only the velocity controller, we applied the well-known admittance control law [7] to transfer the desired torque obtained the proposed controller to the desired velocities for the mobile base.

B. Experimental Results

1) Picking up an object in the box: To validate the proposed whole-body controller with the task transition algorithm, we designed an experimental scenario in which the robot picks up an object in a box. To achieve this scenario, three tasks are defined as follows: $T_{\text{move}} \in \mathbb{R}^6$ for moving the end-effector to the box in the task space, $T_{\text{init}} \in \mathbb{R}^7$ to maintain the initial posture in the joint space, and $T_{\text{grasp}} \in \mathbb{R}^7$ to generate the joint posture for picking up an object. Based on these tasks, the robot performs the SoTs with $T_{\text{move}} \prec T_{\text{init}}$ to reach the box for 0 sec to 20 sec. Then, $T_{\text{grasp}}$ is added in the SoTs as the highest priority using (13) for 20 sec to 30 sec. Finally, the SoTs, $T_{\text{grasp}} \prec T_{\text{move}} \prec T_{\text{init}}$ changes to $T_{\text{move}} \prec T_{\text{init}}$ for 40 sec to 50 sec to pick up an object.

Fig. 8 shows the experimental results of the experiment. As shown in Fig. 8(a), the mobile manipulator could pick up the red object by using the proposed control framework. In particular, thanks to the activation parameter as shown in Fig. 8(b), the mobile manipulator handled the dynamically changing SoTs without the discontinuity of the control inputs (see Fig. 8(c)).

2) Swapping multi-prioritized tasks: Similar to the experiment in the Sec. III-B.1, the experiment for validating the performance for swapping multi-prioritized tasks of the proposed controller was conducted with three tasks, as follows: $T_{ee} \in \mathbb{R}^6$ for moving the end-effector to 20 cm in the x-direction, $T_{\text{mobile}} \in \mathbb{R}^2$ to keep the mobile base at the origin, and $T_{\text{init}} \in \mathbb{R}^7$ for maintaining the initial posture. With these tasks, we designed temporal sequences: $[T_{\text{init}} \prec T_{\text{mobile}} \prec T_{ee} \rightarrow T_{\text{init}} \prec T_{ee} \prec T_{\text{mobile}}]$ for 0 sec to 6 sec, $[T_{\text{init}} \prec T_{ee} \prec T_{\text{mobile}} \rightarrow T_{\text{mobile}} \prec T_{ee} \prec T_{\text{init}}]$ for 12 sec to 17 sec, and $[T_{\text{mobile}} \prec T_{ee} \prec T_{\text{init}} \rightarrow T_{\text{mobile}} \prec T_{\text{init}}]$ for 20 sec to 26 sec.

Fig. 9 shows the experimental results for swapping multi-prioritized tasks. When the robot performed the first transition, the 2-norm error of $T_{\text{mobile}}$ increased because this task is moved to the lowest priority in the SoTs. Next during the second transition, the error of $T_{\text{mobile}}$ decreased, while the error of $T_{\text{init}}$ increased smoothly. This is because the proposed controller swapped the priorities between $T_{\text{mobile}}$ and $T_{\text{init}}$. Finally, after finishing the third transition, the non-holonomic mobile manipulator was located at the origin with maintaining the initial posture of $T_{\text{init}}$. Fig. 9(b) shows the snapshots of the experiments for rearranging tasks.

Consequently, the proposed controller for non-holonomic mobile manipulator can deal with complex tasks with smooth transitions.

IV. CONCLUSIONS

We have presented a novel prioritized whole-body controller with the continuous task transition strategy for a non-holonomic mobile manipulator. Based on the HQP framework, the whole-body controller can execute the predefined SoTs including both equality and inequality tasks. Moreover, with the proposed continuous task transition algorithm, our
controller can treat dynamically changing SoTs without the discontinuous control inputs. We conducted various experimental scenario to demonstrate the performance of the proposed framework by using the four-wheeled mobile base with the 7-DoF robot manipulator. Our future works will involve the extension of the proposed framework for controlling humanoid robots in human-centered environment.

REFERENCES


