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Vertical COM Motion Generation to Reduce Slipping and Mechanical Work during Walking

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Abstract— **A robot foot can slip when the horizontal shear force acting on the foot exceeds the frictional force between the foot and the ground. In the linear inverted-pendulum (LIP) model, the vertical height of the center of mass (COM) is kept constant, and the vertical force is always equal to the gravitational force at any walking speed. However, the horizontal force increases upon increasing the walking speed. This restriction on the vertical force in the LIP model can cause the robot foot to slip at fast walking speeds, as the horizontal force can exceed the frictional force, which is proportional to the vertical force. In this study, we present an optimization method to generate vertical COM motion to maintain the utilized coefficient of friction (uCOF) less than the available coefficient of friction between the foot and the ground, and to minimize the mechanical work of the COM. Vertical motions at various speeds are generated using the proposed optimization method. Subsequently, the generated COM motion patterns are used as reference trajectories of the COM in robot simulation. Optimization and simulation results demonstrate that the mechanical work and uCOF decrease because of the vertical motion. This study suggests a way to generate slip-safe and energy-efficient COM patterns, which, in turn, overcome the limitations of the LIP model by adding vertical COM motion.**

I. INTRODUCTION

Robot-foot slippage is one of the factors responsible for the increasing instability of humanoid robots during walking. It occurs when the horizontal shear force of the supporting foot becomes greater than the friction force between the foot and the ground [1]. To predict the potential for a slip, studies on the relationship between the available coefficient of friction (aCOF) and the utilized coefficient of friction (uCOF) have been conducted in the field of biomechanics [2]. The aCOF is both the static and dynamic coefficient of friction between two objects in contact, and it depends on the properties of the objects [2]. The uCOF is the ratio of the horizontal shear force to the vertical force applied by the supporting foot [3]. Foot slippage occurs during walking when the uCOF exceeds the aCOF between the foot and the ground. For a walking robot, the possibility of a slip depends upon how the horizontal shear force and vertical force both acting on the foot are designed.

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The motion of the center of mass (COM) during walking is often represented using the inverted-pendulum (IP) model. Because the dynamics of the IP model is nonlinear, it is mathematically complicated to generate the COM pattern by using this model. Therefore, the linear IP (LIP) model is widely used to generate the COM pattern of humanoid robots during walking [4]. For mathematical simplification, the LIP model restricts the vertical height of the COM and also requires the orbital energy to be constant [4]. For stable walking, the zero-moment point (ZMP) is controlled to be kept on the supporting foot, following which the COM pattern is generated based on the ZMP pattern [5]. Alternatively, in the LIP model, the capture point (CP) [6], which has the same dynamics as that of the extrapolated COM (XcoM) [7], [8], is used to generate stable COM patterns of humanoid robots during walking.

In the LIP model, the vertical ground reaction force is equal to the gravitational force [4]. However, upon increasing the walking speed, the horizontal ground reaction force increases in proportion with the forward and lateral accelerations of the COM. This increase in the horizontal ground reaction force, while the vertical ground force is being constant, suggests that the uCOF becomes greater than the aCOF at a certain walking speed. Therefore, the robot-foot slippage can occur because of the restriction of the vertical motion by the LIP-model constraints.

By generating the appropriate vertical motion, we can reduce the robot-foot slippage during walking. Various types of vertical motion can set the maximum value of the uCOF to be less than the aCOF between the foot and floor. One of the simple and energy-efficient methods is to minimize the mechanical work of the COM by introducing added vertical motion. Therefore, the COM pattern would become more energy efficient by exchanging kinetic energy and potential energy.

This study aims to generate the appropriate vertical motion of the COM both to reduce the possibility of slipping and to minimize the mechanical work of the COM motion during walking. A robot is simulated to analyze the generated COM motion and to investigate both the mechanical work of the robot joints and the uCOF acting on the robot foot. The paper is organized as follows: In Section II, XcoM and COM trajectories are generated for forward and lateral COM motions. In Section III, an optimization method is proposed to generate vertical COM motion based on the forward and lateral COM motions. Section IV presents the results by drawing comparison between the uCOF and mechanical work of the COM patterns without vertical motion and those

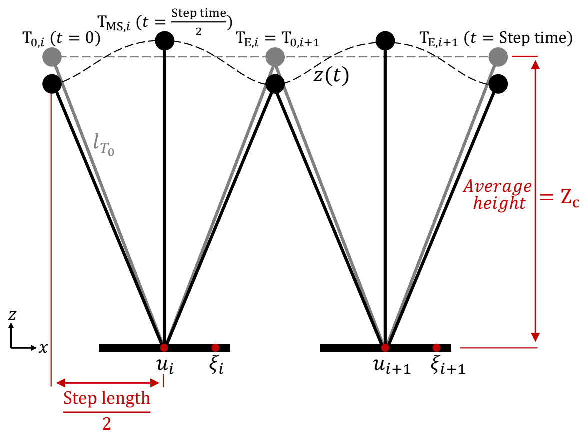

Fig. 1. Vertical-height determination. Forward and lateral positions of the COM are assumed to be located at the middle of the left and right foot, respectively, at step change, $\mathrm{T_0}(t=0)$, to calculate the *average height*, $\mathrm{Z_c}$.

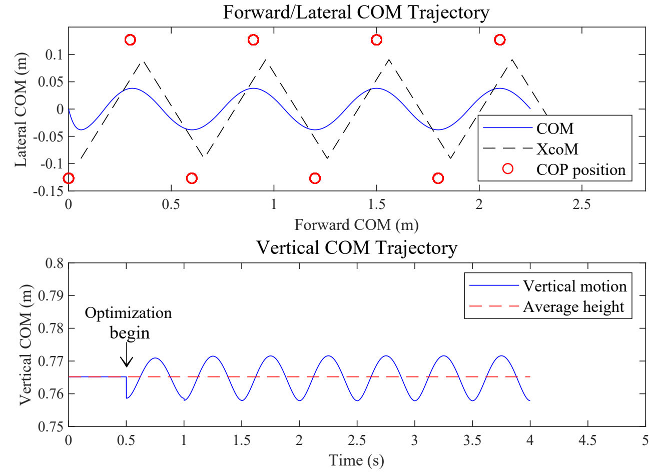

Fig. 2. Forward and lateral trajectories of the COM are generated using the dynamics of the XcoM (Top). The vertical trajectory of the COM is generated using an optimization method given in Section III. C. (Bottom).

with vertical motion.

II. WALKING PATTERN GENERATION

The XcoM in [7], [8] and CP in [6] have been introduced using different notations in the fields of biomechanics and robotics, respectively. However, the dynamics of both the XcoM and CP are the same. In this study, the notation defined in [8] is mainly used, as we use the method presented in [8] to select the initial XcoM.

A. XcoM Dynamics

The position of the XcoM, ξ, is defined as

$$\xi = x + \frac{\dot{x}}{\omega_0}, \tag{1}$$

where x denotes the position of the COM, $\dot{x}$ the velocity of the COM, and ω_0 the eigenfrequency of the inverted pendulum ($\omega_0 = \sqrt{g/\mathrm{Z_c}}$, where $\mathrm{Z_c}$ is *average height* of vertical COM and is introduced later in Section II. B.). Differentiating (1) with respect to time gives

$$\dot{\xi} = \dot{x} + \frac{\ddot{x}}{\omega_0}. \tag{2}$$

The following is the dynamics of the LIP model:

$$\ddot{x} = {\omega_0}^2(x - \mathrm{u}), \tag{3}$$

where u denotes the position of the center of pressure (COP). Substituting (1) and (3) into (2), the XcoM dynamics is given by

$$\dot{\xi} = \omega_0(\xi - \mathrm{u}). \tag{4}$$

The analytic solution of (4), when u is held constant, is given by

$$\xi(t) = (\xi_0 - \mathrm{u})e^{\omega_0 t} + \mathrm{u}. \tag{5}$$

From (1), the analytic solution of the COM dynamics is given by

$$x(t) = (x_0 - \mathrm{u})e^{-(\omega_0 t)} + (\xi_0 - \mathrm{u})\mathrm{sinh}(\omega_0 \mathrm{t}) + \mathrm{u}. \tag{6}$$

Equations (5) and (6) are used to generate both the forward and lateral trajectories of the COM; thus, the equations for lateral XcoM and COM trajectories are not mentioned separately.

B. Vertical-Height Determination

In steady-state walking, it is assumed that the step length and step width are kept constant, and that the forward and lateral motions are symmetric to the midpoint of a step, $\mathrm{T_{MS}}\,(t = \frac{\text{Step time}}{2})$ (see Fig. 1). Furthermore, the forward position of the COM, $x_{\mathrm{T_0}}$, and the lateral position of the COM, $y_{\mathrm{T_0}}$, are located at the middle of COP positions of the left foot and right foot, respectively, at step change, $\mathrm{T_0}(t=0)$. The leg length at the step change, $l_{\mathrm{T_0}}$ should be smaller than the maximum leg length, $l_{\max}$. By specifying $l_{\mathrm{T_0}}$, the vertical height of the COM, $z_{\mathrm{T_0}}$, is calculated as follows:

$$z_{\mathrm{T_0}} = \sqrt{l_{\mathrm{T_0}} - (x_{\mathrm{T_0}} - \mathrm{u_x})^2 - (y_{\mathrm{T_0}} - \mathrm{u_y})^2}, \tag{7}$$

where

$$x_{\mathrm{T_0}} - \mathrm{u_x} = \frac{\text{Step length}}{2}, \quad y_{\mathrm{T_0}} - \mathrm{u_y} = \frac{\text{Step width}}{2}.$$

The term $z_{\mathrm{T_0}}$ obtained from (7) is defined as the *average height*of the COM, $\mathrm{Z_c}$.

C. Initial XcoM Planning

According to [8], the relationship between the initial XcoM and COP of the foot at the i^{th} step is obtained as follows:

$$\xi_{0,i} = \mathrm{u}_i + \mathrm{b_{x,y}}, \tag{8}$$

where

$$\begin{cases} \mathrm{b_x} = \frac{\text{Step length}}{(e^{(\omega_0 \text{Step time})}-1)} & \textit{(Forward margin of stability)} \\ \mathrm{b_y} = \frac{(-1)^i \text{Step width}}{(e^{(\omega_0 \text{Step time})}+1)} & \textit{(Lateral margin of stability).} \end{cases}$$

The sign of b_y is changed according to the left foot or right foot; thus, i is used to alter the sign. It is assumed that the supporting foot is in full contact with the ground during the stance phase. Furthermore, it is assumed that u_i is positioned on the center of the supporting foot and is constant.

D. Forward and Lateral COM-Motion Generation

The forward and lateral trajectories of both the XcoM and COM are generated using (5) and (6) with the desired step length, step width, step time, ω_0, $\xi_{0,1}$, and the initial positions both of the COM and COP, $x_{\mathrm{T_0},1} = 0$, $y_{\mathrm{T_0},1} = 0$, $u_{x,1} = 0$, $u_{y,1} = \frac{\text{Step width}}{2}$. The XcoM and COM at the end of a step, $\mathrm{T}_{\mathrm{E},i}(t = \text{Step time})$, become the initial positions of both the XcoM and COM for the next step ($\xi_{\mathrm{T_0},i+1} = \xi_{\mathrm{T_E},i}$, $x_{\mathrm{T_0},i+1} = x_{\mathrm{T_E},i}$). Subsequently, the COP positions of the foot for the next step, u_{i+1}, are obtained using (8). Fig. 2 (Top) depicts an example of the forward and lateral trajectories of the COM with the desired COP positions.

Double-support period, $\mathrm{T_{DSP}}$, is assigned to interpolate the velocity trajectories of the COM by using the cubic spline curve during the period from $\mathrm{T}_{\mathrm{E},i} - \frac{\mathrm{T_{DSP}}}{2}$ to $\mathrm{T}_{0,i+1} + \frac{\mathrm{T_{DSP}}}{2}$. The double-support time is calculated using (9), which is a regression equation based on human-walk-related data to estimate the single-support time ($\mathrm{T_{SSP}}$) and double-support time ($\mathrm{T_{DSP}}$) from step time ($\mathrm{T_{Step}}$) [9]. We have

$$\mathrm{T_{SSP}} = \alpha \cdot (2 \cdot \mathrm{T_{Step}}) + \beta, \quad \mathrm{T_{DSP}} = \mathrm{T_{Step}} - \mathrm{T_{SSP}}, \quad (9)$$

where α is 0.2070 and β is 0.1782.

III. VERTICAL-MOTION OPTIMIZATION

A. uCOF Calculation

The uCOF is defined as follows:

$$uCOF = \frac{\sqrt{F_{\text{forward}}^2 + F_{\text{lateral}}^2}}{F_{\text{vertical}}}, \quad (10)$$

where F_{forward}, F_{lateral}, and F_{vertical} denote the forward, lateral, and vertical ground reaction forces, respectively, acting on the supporting foot during walking. However, considering the dynamics of the LIP model, only the horizontal COM acceleration and the gravitational acceleration act on a robot foot. Therefore, the $uCOF_{\mathrm{LIPM}}$ is calculated as follows:

$$uCOF_{\mathrm{LIPM}} = \frac{\sqrt{\ddot{x}^2 + \ddot{y}^2}}{g}. \quad (11)$$

Conversely, upon assuming that the vertical acceleration of the COM occurs because of its vertical motion, $uCOF_{\mathrm{VM}}$ is calculated as follows:

$$uCOF_{\mathrm{VM}} = \frac{\sqrt{\ddot{x}^2 + \ddot{y}^2}}{\ddot{z} + g}, \quad (12)$$

where $\ddot{x}, \ddot{y}$, and $\ddot{z}$ denote the forward, lateral, and vertical accelerations of the COM, respectively. $\ddot{x}$ and $\ddot{y}$ are obtained by differentiating (6). $\ddot{z}$ is obtained by differentiating (16), which is introduced later in Section III. C.

Fig. 3 depicts the change in the uCOF as vertical acceleration of the COM changes, where the forward and lateral accelerations are identical. The maximum uCOF decreases when the COM begins to accelerate vertically.

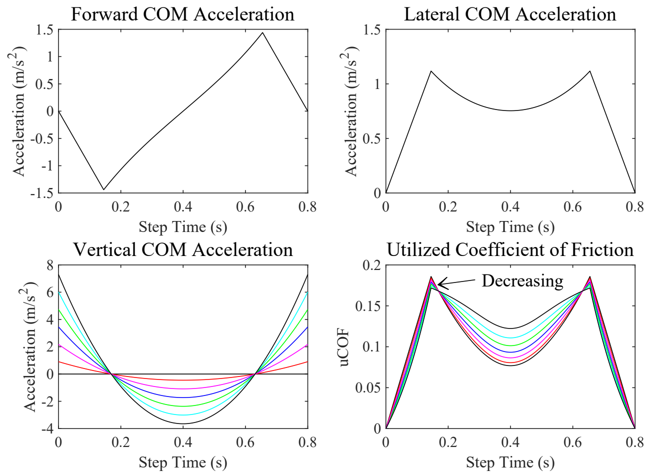

Fig. 3. uCOF depending on vertical acceleration. uCOF is calculated as the ratio of the horizontal shear force to the vertical force by using (12). The maximum uCOF decreases when the COM begins to accelerate vertically with identical forward and lateral accelerations.

B. COM Mechanical Energy and Power Calculation

The total energy of the COM in the IP model is conserved by the interchange of the kinetic energy and the potential energy. However, in the LIP model, no kinetic energy exists in the vertical direction, and the potential energy remains constant. In the LIP model, the total energy of the COM, E_{LIPM}, and the total energy due to the vertical motion, E_{VM}, are given as follows:

$$E_{\mathrm{LIPM}} = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}m\dot{y}^2 + mg\mathrm{Z_c} \quad (13)$$

and

$$E_{\mathrm{VM}} = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}m\dot{y}^2 + \frac{1}{2}m\dot{z}^2 + mgz, \quad (14)$$

where $\dot{x}$, $\dot{y}$, and $\dot{z}$ denote the forward, lateral, and vertical velocities, respectively. $\mathrm{Z_c}$ denotes the *average height*, while z denotes the vertical trajectory of the COM over time; the vertical trajectory of the COM is explained in (16).

The mechanical power of the COM is the rate of the total energy consumption. On differentiating (14), the mechanical power of the COM undergoing vertical motion, P, is obtained as follows:

$$P = m\dot{x}\ddot{x} + m\dot{y}\ddot{y} + m\dot{z}\ddot{z} + mg\dot{z}. \quad (15)$$

To obtain the mechanical power when the COM does not undergo vertical motion, only the terms representing the forward and lateral motions remain in (15).

C. Vertical COM-Motion Generation

At step change, $\mathrm{T_0}$, the forward and lateral velocities of the COM are fastest, and, consequently, the kinetic energy is highest at that time. From the law of energy conservation, the potential energy should be lowest at the point of highest kinetic energy. Furthermore, in the middle of a step, $\mathrm{T_{MS}}$, the forward and lateral velocities of the COM are the slowest, and, consequently, the kinetic energy is the lowest. Therefore, the potential energy should be highest at $\mathrm{T_{MS}}$. Based on the

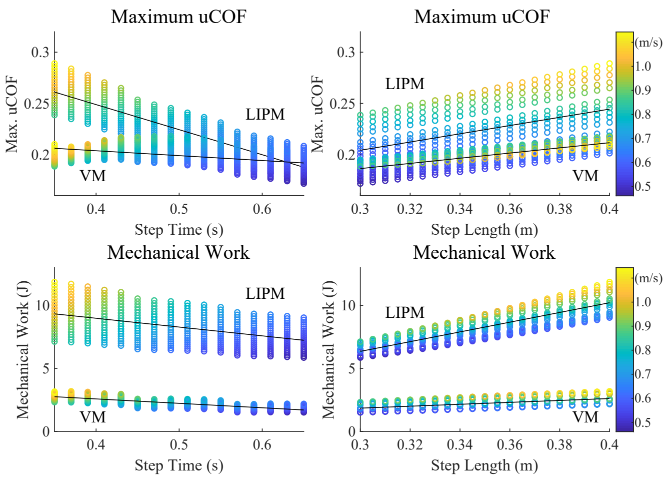

Fig. 4. uCOF and positive mechanical work. Colored circles represent the maximum uCOF and positive mechanical work as speed changes (the color bar in the right side of the graph) at given step times or step lengths. The black lines represent the trend lines for the maximum uCOF and positive mechanical work. The maximum uCOF (Top) and the positive mechanical work (Bottom) decrease when there is vertical motion.

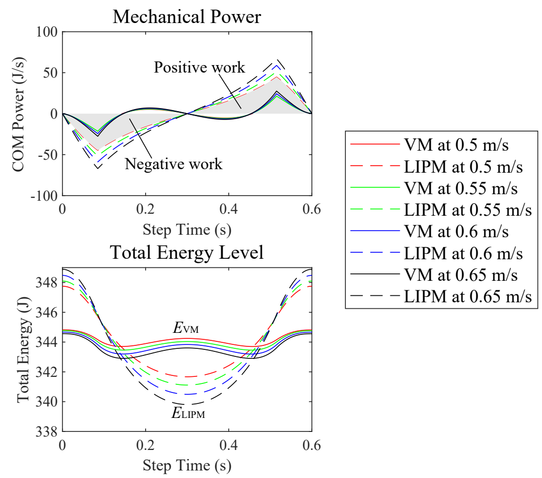

Fig. 5. Mechanical power and total energy level. Positive work of the COM decreases under the vertical motion (Top). The change in the total energy under vertical motion is smaller than that without the vertical motion (Bottom).

change in the potential energy, the vertical motion may have the lowest point at $\mathrm{T_0}$ and the highest point at $\mathrm{T_{MS}}$. Fig. 1 depicts such a vertical motion. In addition, the vertical motion should meet four equality constraints and two inequality constraints in (17). Then, a 4^{th}-order polynomial is the smallest order polynomial satisfying the aforementioned constraints. To simplify the problem, the following 4^{th}-order polynomial representing the vertical COM trajectory is used:

$$z(t) = p_1 t^4 + p_2 t^3 + p_3 t^2 + p_4 t + p_5. \qquad (16)$$

An optimization problem is established to determine the coefficients, $p = \{p_1, p_2, p_3, p_4, p_5\}$ of the polynomial in (16). When the increments or decrements in the kinetic energy of the forward and lateral motions are totally exchanged to the potential energy and kinetic energy of the vertical motion, the positive work, W_+, becomes zero by the law of energy conservation. The optimization problem can be formulated as follows:

$$\begin{aligned}
\min_{p} \quad & W_+ = \int_{\mathrm{T_0}}^{\mathrm{T_E}} P_+ dt, \\
\text{s. t.} \quad & z_i(\mathrm{T_0}) = z_i(\mathrm{T_E}) \\
& \dot{z}_i(\mathrm{T_0}) = \dot{z}_i(\mathrm{T_E}) \\
& \ddot{z}_i(\mathrm{T_0}) = \ddot{z}_i(\mathrm{T_E}) \\
& \mathrm{average}(z_i(t)) = \mathrm{Z_c} \\
& \max(l_i(t)) \le l_{\max} = 0.8 \\
& \max(uCOF_{\mathrm{VM},i}(t)) \le aCOF = 0.22
\end{aligned} \qquad (17)$$

where P_+ denotes the positive power, which is the mechanical power having only positive value. The mechanical power is calculated in (15).

The four equality constraints are necessary to satisfy the symmetry of the trajectory, and the two inequality constraints to satisfy the physical limitations such as the maximum leg length of a robot and the aCOF between the foot and the ground. Furthermore, the lower and upper boundaries of the polynomial coefficients were set to $p_{\mathrm{lower}} = \{0, -25, 0, -0.1, 0.6\}$ and $p_{\mathrm{upper}} = \{25, 0, 10, 0.1, 0.8\}$, respectively.

In this study, $l_{\max}$ is set to be 0.8 m considering the physical limitation of the humanoid robot, DYROS-JET, which is used for simulation in Section IV. C. The aCOF is set to 0.22 considering the value of the aCOF investigated in other studies [10], [11], [12]. According to [10], [11], the aCOF is generally higher than 0.4 for various floor conditions (wet, dry, and clayey) and shoe materials (rubber, neolite). In [12], it was shown that the slippage probability was 10^{-6} at the aCOF of 0.3 for human walking. In this study, we aim to obtain the maximum uCOF of less than 0.22 considering the lower aCOF. Lower aCOF are associated with slippery surfaces such as floors contaminated by water or oil.

Global Optimization Toolbox in *MATLAB (MathWorks, Inc.)* was used for optimization. *MultiStart* function was used to find the global minimum with a local solver, *fmincon* function, applying the *SQP* algorithm at multiple start points. Fig. 2 (Bottom) depicts an example of vertical trajectory of the COM at a given *average height*. The vertical trajectories obtained from the optimization were used as reference trajectories in robot simulation.

IV. EFFECT OF THE VERTICAL MOTION

A. Change in uCOF and Mechanical Work

Fig. 4 depicts the maximum $uCOF$ and positive mechanical work derived using the optimization in Section III.

The $uCOF_{\mathrm{LIPM}}$ increases significantly as the speed increases (the color bar in the right side of the graph), either by shorter step times or by longer step lengths. The $uCOF_{\mathrm{VM}}$ also increases as the speed increases, but the increase is much smaller than that in the $uCOF_{\mathrm{LIPM}}$ (see Fig. 4. (Top)). The $uCOF_{\mathrm{LIPM}}$ undergoes a steep linear increase as the speed increases, suggesting that the potential for a slip becomes high at a fast walking speed.

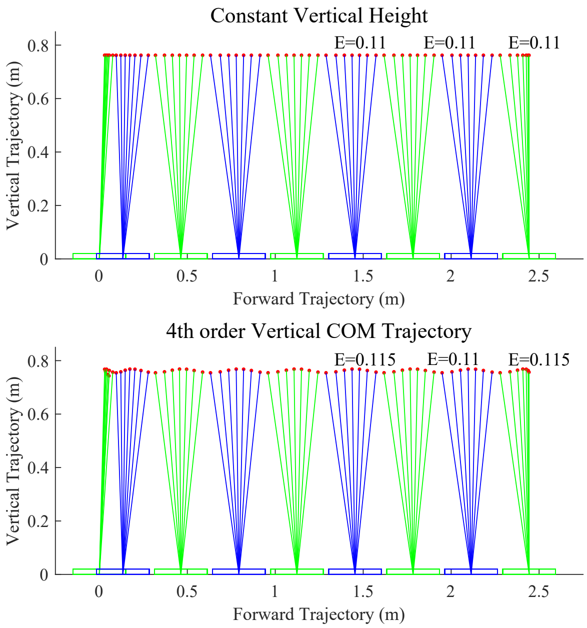

Fig. 6. COM trajectory on the sagittal plane. Orbital energy without the vertical motion (Top) and with the vertical motion (Bottom) are presented. The trajectories were obtained at the following conditions: step length = 0.33 m, step time = 0.51 s, and *average height* = 0.762 m.

The positive work of the COM also increases as the speed increases. Fig. 4. (Bottom) indicates that the positive work of the COM decreases significantly under the vertical motion compared to that without the vertical motion.

Fig. 5. (Top) depicts that the negative work during the first half of a step is reduced because of the vertical motion of the COM. Therefore, the required positive work also decreases during the last half of the step. As the speed increases, both the negative work and the positive work become large in the case of the motion of the LIP model. However, the negative work and the positive work of the COM under the vertical motion are less affected by the change in speed (see Fig. 5. (Top)).

The change in the total energy of the COM under the vertical motion, E_{VM}, is significantly smaller than that of the LIP model, E_{LIPM}. The total energy under the vertical motion is fairly constant (see Fig. 5. (Bottom)).

B. Orbital Energy Error

The orbital energy, E_{orbital}, of the COM trajectory obtained using the LIP model is constant [4]. Adding the vertical motion means E_{orbital} is no longer constant. In this section, to investigate the variation in E_{orbital}, the orbital-energy error is calculated upon adding the vertical motion to the COM trajectory of the LIP model.

The orbital energy is defined as

$$E_{\mathrm{orbital}} = \frac{1}{2}\dot{x}^2 - \frac{1}{2}\omega_0^2 x^2, \tag{18}$$

where x denotes the displacement from the foot to the COM. The orbital-energy error, E_{error}, when there is vertical motion, is calculated as follows:

$$E_{\mathrm{error}} = \frac{E_{\mathrm{max}} - E_{\mathrm{min}}}{E_{\mathrm{avg}}} 100(\%), \tag{19}$$

where E_{max}, E_{min}, and E_{avg} denote the maximum, minimum, and average of E_{orbital} during a step, respectively.

E_{orbital} was approximately 0.07 J/kg when the velocity was 0.55 m/s, step length 0.305 m, step time 0.55 s, and $\mathrm{Z_c}$ 0.765 m; at this condition, E_{error} was approximately 4.9%. Furthermore, E_{orbital} was approximately 0.11 J/kg when the velocity was 0.65 m/s, step length 0.33 m, step time 0.51 s, and $\mathrm{Z_c}$ 0.762 m; at this condition, E_{error} was approximately 4.5%. Fig. 6 depicts the orbital energy and COM trajectory at 0.65 m/s on the sagittal plane. The amplitude of the vertical motion was approximately 0.016 m. Under the vertical motion, E_{orbital} is not constant; however E_{error} seems to be small.

C. Simulation Results

Based on the obtained COM trajectories in Sections II and III, a humanoid robot, DYROS-JET, was simulated using *V-REP* simulator (*Coppelia Robotics. Co.)* with *Vortex* dynamics engine (*CM Labs. Co.)*. The total body mass and height of the robot were 48 kg and 1.63 m, respectively. The DYROS-JET robot had 32 degrees of freedom (DoFs), which are 8 DoFs for each arm, 6 DoFs for each leg, 2 DoFs for the torso, and 2 DoFs for the head [13]. In this simulation, the joints of the legs were controlled to follow the desired COM and foot trajectories. The joints of the arms and upper body were maintained in the initial position. COM Jacobian based closed-loop inverse kinematics algorithm [14] was adopted to control the joints at the control frequency of 1000 Hz.

Fig. 7 depicts the simulation result of walking with the vertical motion at the speed of 0.55 m/s, step length 0.305 m, and step time 0.55 s. Joint velocity, joint torque, and ground reaction force of the foot were obtained from the simulation. Subsequently, the positive joint work, $W_{J,+}$, was calculated to compare it with the positive COM work, W_{+}, which was calculated in (17).

The mechanical joint power of the supporting leg is calculated as follows:

$$P_{\mathrm{J}} = \sum_{k=1}^{n} P_k = \sum_{k=1}^{n} \dot{q}_k \, \tau_k, \quad n = 6, \tag{20}$$

where P_{J} denotes the sum of joint powers, and P_{k}, $\dot{q}_k$, and τ_k the joint power, joint velocity, and joint torque of the k^{th} joint, respectively. The number of joints, n, is 6 in this study for the supporting leg. The positive joint work, $W_{\mathrm{J},+}$, for the supporting leg is then obtained as follows:

$$W_{\mathrm{J},+} = \int_{\mathrm{T_0}}^{\mathrm{T_E}} P_{\mathrm{J},+} \, dt, \tag{21}$$

where $P_{\mathrm{J},+}$ denotes the positive joint power.

To draw comparison between the $uCOF$(simul.) in robot simulation and the $uCOF$(model) calculated in (11) and (12), the $uCOF$(simul.) was calculated from (10) by using the ground reaction forces obtained from the simulation.

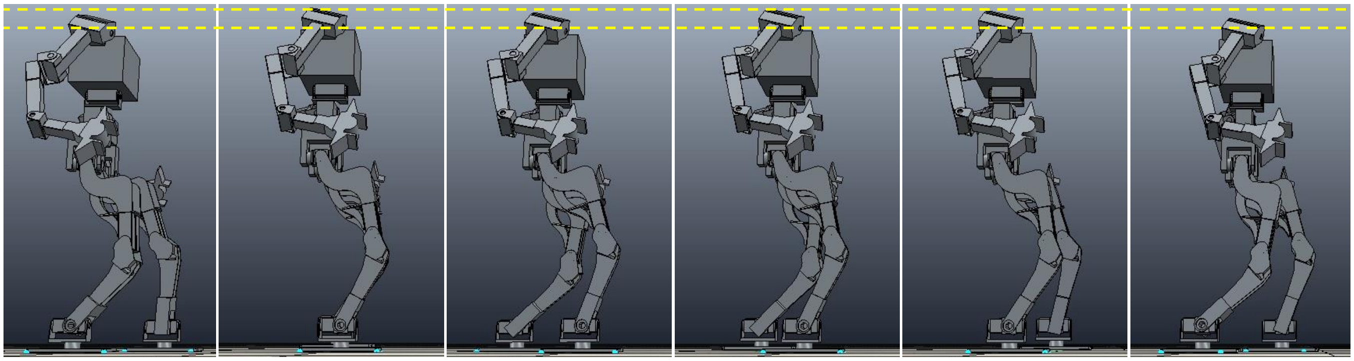

Fig. 7. Robot simulation using *V-REP* simulator. From left to right, the first, third, and sixth pictures depict the robot in the double-support period. The other pictures depict the robot in the single-support period. The vertical height of the robot is changing.

Table 1 presents the maximum $uCOF$(model) calculated using the model, the $uCOF$(simul.) at the end of the double-support phase in the simulation, and W_+ and $W_{J,+}$ during a step. The optimization results demonstrate that the uCOF for the 4^{th}-order vertical motion is lower than that without vertical motion. The simulation results also show that $W_{J,+}$ decreases significantly under vertical motion compared to that without the vertical motion.

TABLE I

COMPARISON OF MODEL AND SIMULATION RESULTS

Speed (m/s)	Z_c (m)	$z_{(t)}$ method	$uCOF$ (model)	$uCOF$ (simul.)	W_+ (J)	$W_{J,+}$ (J)
0.55	0.765	constant	0.1960	0.1989	6.42	7.60
		4^{th}-order	0.1867	0.1937	1.81	3.00
0.60	0.763	constant	0.2056	0.2055	7.00	7.51
		4^{th}-order	0.1935	0.2009	1.99	3.40
0.65	0.762	constant	0.2129	0.2165	7.42	8.21
		4^{th}-order	0.1979	0.2050	2.11	4.00

W_+ denotes the positive COM work.
$W_{J,+}$ denotes the positive joint work in robot simulation.

V. CONCLUSIONS

This study aimed to generate the vertical motion of the COM to reduce the possibility of robot-foot slippage and to minimize the positive mechanical work during walking. Section III. C presented an optimization method to create vertical motion by minimizing the positive mechanical work of the COM motion within the constraints for the uCOF. Using the presented method, not only the COM motion became more stable by reducing the potential for a slip, but also the COM motion became energy efficient. Upon increasing the walking speed, the forward and lateral accelerations became large, thereby requiring greater vertical force for the foot not to slip. Because the LIP model keeps the vertical height constant, another trajectory-generation method, such as the IP model or the SLIP model, is necessary to reduce foot slippage. The approach in this study suggested a way to overcome the aforementioned limitation of the LIP model by adjusting the COM trajectory by using the concept of energy exchanging. This method may be applicable to other situations in which vertical-force generation is required during walking. For future work, the walking experiment will be conducted on a real humanoid robot, DYROS-JET.

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